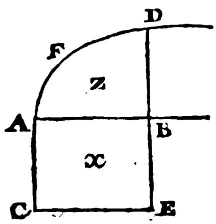
## PROB. IX.

### *To determine the Area of any Curve proposed.*

1. The resolution of the Problem depends upon this, that from the relation of the Fluxions being given, the relation of the Fluents may be found, (as in Prob. 2.) And first, if the right Line BD, by the motion of which the Area required AFDB is described, move upright upon an Absciss AB given in position, conceive (as before) the Parallelogram ABEC to be described in the mean time on the other side AB, by a line equal to unity. And BE being suppos'd the Fluxion of the Parallelogram, BD will be the Fluxion of the Area required.



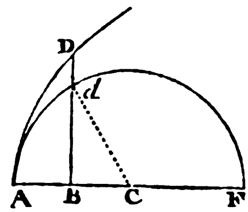

2. Therefore make AB $= x$, and then also ABEC $= 1 \times x = x$, and BE $= \dot{x}$. Call also the Area AFDB $= z$, and it will be BD $= \dot{z}$, as also $= \frac{\dot{z}}{\dot{x}}$, because $\dot{x} = 1$. Therefore by the Equation expressing BD, at the same time the ratio of the Fluions $\frac{\dot{z}}{\dot{x}}$ is

is exprefs'd, and thence (by Prob. 2. Cafe 1.) may be found the relation of the flowing quantities $x$ and $z$.

3. Ex. 1. When BD, or $\dot{z}$, is equal to fome fimple quantity.

4. Let there be given $\frac{xx}{a} = \dot{z}$, or $\frac{\dot{z}}{\dot{x}}$, (the Equation to the Parabola,) and (Prob. 2.) there will arife $\frac{x^3}{3a} = z$. Therefore $\frac{x^3}{3a}$, or $\frac{1}{3}$ AB × BD, = Area of the Parabola AFDB.

46. Let the Hyperbola AD be proposed, whose Equation is $\sqrt{x+xx}=z$; its Vertex being at A, and each of its Axes is equal to Unity. From what goes before, its Area ADB $= \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{5}x^{\frac{5}{2}} - \frac{1}{28}x^{\frac{7}{2}} + \frac{1}{72}x^{\frac{9}{2}} - \frac{5}{704}x^{\frac{11}{2}}$, &c. that is $x^{\frac{1}{2}}$ into $\frac{2}{3}x + \frac{1}{5}x^2 - \frac{1}{28}x^3 + \frac{1}{72}x^4 - \frac{5}{704}x^5$, &c. which Series may be infinitely produced by multiplying the last term continually by the succeeding terms of this Progression $\frac{1.3}{2.5}x. \ \frac{-1.5}{4.7}x. \ \frac{-3.7}{6.9}x. \ \frac{-5.9}{8.11}x. \ \frac{-7.11}{10.13}x.$ &c. That is, the first term $\frac{2}{3}x^{\frac{3}{2}} \times \frac{1.3}{2.5}x$ makes the second term $\frac{1}{5}x^{\frac{5}{2}}$: Which multiply'd by $\frac{-1.5}{4.7}x$ makes the third term $-\frac{1}{28}x^{\frac{7}{2}}$: Which multiply'd by $\frac{-3.7}{6.9}x$ makes $\frac{1}{72}x^{\frac{9}{2}}$ the fourth term; and so *ad infinitum*. Now let AB be assumed of any length, suppose $\frac{1}{4}$, and writing this Number for $x$, and its Root $\frac{1}{2}$ for $x^{\frac{1}{2}}$, and the first term $\frac{2}{3}x^{\frac{3}{2}}$ or $\frac{2}{3} \times \frac{1}{8}$, being reduced to a decimal Fraction, it becomes 0.083333333, &c. This into $\frac{1.3}{2.5.4}$ makes 0.00625 the second term. This into $\frac{-1.5}{4.7.4}$ makes $-$ 0.0002790178, &c. the third term. And so on for ever. But the terms, which I thus deduce by degrees, I dispose in two Tables; the affirmative terms in one, and the negative in another, and I add them up as you see here.

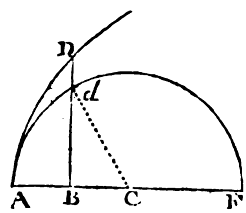



+0.

| | |
|---|---|
| + 0.08333333333333333 | — 0.0002790178571429 |
| 62500000000000 | 34679066051 |
| 271267361111 | 834465027 |
| 5135169396 | 26285354 |
| 144628917 | 961296 |
| 4954581 | 38676 |
| 190948 | 1663 |
| 7963 | 75 |
| 352 | 4 |
| 16 | — 0.0002825719389575 |
| 1 | + 0.0896109885646518 |
| + 0.0896109885646518 | 0.0893284166257043 |

Then from the ſum of the Affirmatives I take the ſum of the negatives, and there remains 0.0893284166257043 for the quantity of the Hyperbolic Area ADB; which was to be found.

47. Now let the Circle A*d*F be propoſed, which is expreſſed by the equation $\sqrt{x-xx}=z$; that is, whoſe Diameter is unity, and from what goes before its Area A*d*B will be $\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{5}x^{\frac{5}{2}}-\frac{1}{28}x^{\frac{7}{2}}-\frac{1}{72}x^{\frac{9}{2}}$, &c. In which Series, ſince the terms do not differ from the terms of the Series, which above expreſs'd the Hyperbolical Area, unleſs in the Signs + and —; nothing elſe remains to be done, than to connect the ſame numeral terms with other ſigns; that is, by ſubtracting the connected ſums of both the afore-mention'd tables, 0.0898935605036193 from the firſt term doubled 0.1666666666666, &c. and the remainder 0.0767731061630473 will be the portion A*d*B of the circular Area, ſuppoſing AB to be a fourth part of the diameter. And hence we may obſerve, that tho' the Areas of the Circle and Hyperbola are not compared in a Geometrical conſideration, yet each of them is diſcover'd by the ſame Arithmetical computation.




48. The portion of the circle A*d*B being found, from thence the whole Area may be derived. For the Radius *d*C being drawn, multiply B*d*, or $\frac{1}{4}\sqrt{3}$, into BC, or $\frac{1}{4}$, and half of the product $\frac{1}{32}\sqrt{3}$, or 0.0541265877365275 will be the value of the Triangle C*d*B; which added to the Area A*d*B, there will be had the Sector AC*d* = 0.1308996938995747, the ſextuple of which 0.7853981633974482 is the whole Area.

49. And

49. And hence by the way the length of the Circumference will be 3.14159265358979 28, by dividing the Area by a fourth part of the Diameter.